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RADIO INTERFEROMETRY INSTRUMENTATION: A REFRESHER COURSE

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Preface

The solar telescope array - BDA, is being set up by the DAS group of the INPE, SJC, Brazil, with participation from astronomers and engineers from UC Berkeley, NCRA-TIFR and IIA, India. During my stay in Brazil during 2002-2003, I was asked to conduct a course of lectures on practical aspects of radio interferometry, by Prof. H. S. Sawant and share my experience in building radio interferometers like the GMRT, India. The lectures are primarily refresher materials, with emphasis on basic and practical concepts of radio interferometry. I have attempted to explain the details with numerical examples where-ever possible. It is hoped that the material will help the users, in day to day analysis in system engineering and operations of the instrument.

I whole heartedly thank Prof. Sawant and his team for providing this opportunity.

M. R. Sankararaman

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CONTENTS

The refresher course is intended for scientists and engineers who are fairly familiar with the art and science of Radio Astronomy (RA) but need some retuning of the receiver system concepts needed for Radio Interferometry. Importance has been given to system analysis and numerical illustrations. A list of references can be found at the end of the course material.

The organization of the modules is as follows:

- Introduction: types of Radio Telescopes (RT) - Polar mount, Alt-Az type, celestial and terrestrial co-ordenates systems and conversion of HA-Dec to Alt-Az coordenates
- RA signals, power flux density, spectral power density, brightness temperature, antenna beam width, resolving power
- Primary and secondary beam patterns, antenna gain, noise temperature, G/T concepts, computation of noise figure of the Rx. system, parabolic dish characteristics
- Sensitivity calculation for a Total Power system, limitations of TP system, Dicke receiver, need for interferometry
- Basic interferometer, time and phase delay, group delay and phase delay concepts, fringe formation, fringe frequency, fringe rate calculation
- Effective area of interferometer, mathematical cross correlator o/p for passband signals, sensitivity and S/N of a correator interferometer
- Noise in the receiver system, noise sources, grounding and shielding

1 INTRODUCTION

The advantages of a fully steerable antenna need not be emphasized here. There are two ways of mounting RTs namely, polar mount and equatorial mount.

1.1 POLAR MOUNT

Here, the axis of the telescope is made to be parallel to the axis of rotation of the earth. The the E-W motion follows the source in HA and rotates with the speed of rotation of the earth. The N-S tracking of the source is done either by rotating the telescope mechanically or by steering the antenna beam electronically as per the Dec coordinate of the source. The Ooty Radio Telescope (ORT) is a polar mount antenna, having mechanical (E-W) an electronic (N-S) beam positioning. It consists of a 560 m × 30 m parabolic cylindrical reflector and a linear array of 1060 dipoles at the focus. The telescope has been constructed on a hill slope of 11 deg. which corresponds to the lat. of the place (Ootacamund, India), thereby making the axis of the telescope lie parallel to the N-S rotation axis of the earth (see details in the website www.ncra.tifr.res.in). The beam coverage is about 140° in E-W and 120° in N-S. The beam is steered in N-S by setting the 5 bit phase shifter as per declination. The signals are combined in a X-mas tree fashion. Because of the cost factor and the complex engineering aspects involved in constructing the telescope especially at high latitudes, not many such telescopes of this type have been constructed.

1.2 ALT-AZ MOUNT

This is by far the widely used type of mount for reasons of cost and ease of mounting of the mechanical systems. In order the track a radio source continuously, we need to position the telescope in both axes, from time to time. The position of the radio source in the celestial sphere are referred as Right Ascension and Declination (Figure 1). The necessary conversion from celestial (HA, Dec) to terrestrial coordinate (Alt-Az) system is done as follows (Birney, 1991):

The RA and HA are related by the equation:

HA = sidereal time - RA

Given the Latitude of the place, Dec and HA of the source, the Alt and Az coordinates at an given time is calculated as:

$$\sin(\mathrm{Alt}) = \sin(\mathrm{Lat}) \,.\, \sin(\mathrm{dec}) + \cos(\mathrm{Lat}) \,.\, \cos(\mathrm{Dec}) \,.\, \cos(\mathrm{HA})$$

$$\sin(\mathrm{Az}) = \sin(\mathrm{HA}) \,.\, \cos(\mathrm{Dec})/\mathrm{Cos}(\mathrm{Alt})$$

The information can also be obtained from the website: http://susdesign.com/sunangle

2 RECEPTION OF RADIO SIGNALS

The emissions from the cosmic radio sources are of random Gaussian noise type. The radiated signals get attenuated in intensity while passing through the intervening medium, due to absorption, scattering and other processes. The wavelengths over which the intervening medium between the observer on earth and the radio sources is essentially transparent to the radio waves, ranging from about 30 m to 1 mm. This natural advantage is exploited by radio astronomers for improving the sensitivity of the RTs and viewing the distant objects. By and larger, the emission from the sources, varies slowly with frequency and is essentially constant over the bandwidth of observation - continuum radiation. In contrast, some sources emit at specific wavelengths too, due to the molecular and atomic processes taking place within them, - spectral line radiation. Typical examples are the classical absorption line of Hydrogen at 1420.406 MHz, OH lines in the 1600-1720 MHz band and so on (Handbook on Radio Astronomy, 1995). The other class of emission is by the pulsars, whose precise rotation makes the atomic clocks feel shy.

We define the flux S a brightness B of the source as follows: The power received from a source depends upon its angular extent w.r.t the antenna beam, the bandwidth of the receiver $\delta\nu$ and the effective area, A_e, intercepting the radiation and The Brightness B of the source = power/unit area/unit bandwidth/steradian and derive the expression for power density S.

The intensity of illumination S and the total received power P are directly related to the beam area and the bandwidth of the receiver as:

$$S = \int (B)\, \Delta\Omega$$

$$P = \int (S)\, \delta\nu$$

The emission mechanisms can be either thermal or non thermal (Synchrotron emission). Accordingly, the spectrum (flux/unit frequency interval) can be of the forms:

$$f = 2\,\pi\,k\,T\,\nu^2/c^2 \text{ (thermal emission)} \tag{1}$$

$$f \propto \nu^{-\alpha} \text{ (non-thermal emission)} \tag{2}$$

where the spectral index ν lies between 0.2 to 1.2.

Let us now see how it works out in the case of a 4 m dish (physical area A = 12.56 m^2, antenna efficiency η = 0.5), operating at a center frequency of 1.4 GHz and having a receiver BW of 2 MHz. Let the source observed be having a flux density (S)100 mJ ($1J = 10^{-26}$ Wm^{-2}Hz^{-1}).

$$P = SA\eta\delta\nu = 0.1 \times 10^{-26}\ .\ 12.56\ .\ 0.5\ .\ 2 \times 10^{6} = 1256 \times 10^{-20} \text{ Watts}$$

It is useful to understand what is field of view of the beam or beam area and resolving power of a RT. The beam area $\Delta\Omega$, is $\approx \theta\phi$, where the two angles represent the HPBW (Half Power Beam Width) in the vertical anf horizontal planes.

The power received is directly proportional to $\Delta\Omega$. If the source is a small in size (point source) w.r.t. the above angles, the antenna is fully illuminated. On the otherhand, if the beam is small, the power/solid angle has to be intergrated over the entire area of the source.

The resolving power defines the angular separation between points at which the first nulls occur (BWFN - Beam Width between First Nulls).

BWFN = 2.122 λ/D, where, D is the aperture of the antenna.

In the case of a 4 m dish, at the wavelength of 0.2 m, the BWFN can be calculated and found to be 7°.

It is worth revisiting the geometry of a parabolic dish.

The parabola is described by the equation: $y = x^2/4F$, which means that unit surfaces on the surface of the dish, redirect the incoming radiation, with varying degrees of inclination depending on their location w.r.t. the vertex, such that they meet at the focus.

If the depth of the dish is denoted by d, $F = D^2/16d$. This relationship can be used for locating the focus, during fabrication.

The Directrix is as far behind as the focus in the front. This property can be used for checking the surface of the reflector and estimating the error. Any point on the surface should be equidistant from the directrix and the focal points. The error is the difference between he measurements. Many such points can be taken to computed the rms surface error. The surface irregularities affect the power collected at the focus in the following manner. A deviation ϵ from the ideal reflector results in a path error of $2\pi\ \epsilon$ (equal to a phase error $\phi = 2\pi\ 2\epsilon/\lambda$). The amplitude and power are reduced by $\cos\phi$ and $\cos^2(\phi)$, respectively.

In general, $\epsilon = \lambda/20$, is taken as an error limit for antenna construction. Under this condition, the power is reduced by $\cos^2(\pi/5) = 0.65$.

The commonly used type of mounting the feed at the focus is either direct (prime focus mount) or by using s secondary reflector to focus the energy towards the center of the dish (Cassegrain mount). The latter is used if the feed element could significantly block the incoming radiation or in situations where the temperature of the front end receiver need to be maintained to great accuracy and so on. The individual polar beam response of the feed is termed as primary pattern, while in the dish, it is called the secondary response. Ideally one would like the beam of the feed cover completely the entire reflector, without picking any radiation form sources outside the reflector. In practice, for a given F/D, the response falls down away from the center of the dish (bore sight). Typically, it is 10 dB below at the rim of the dish. Radiations form outside the dish could be picked up, raising the systems temperature. If the F/D is made smaller to reduce this unwanted response from outside the aperture of the dish, the feed may receive only from part of the dish, thus the aperture efficiency could be reduced. An the feed is mounted for an optimum F/D, between 0.4 to 0.35.

3 GAIN OF THE ANTENNA

Antennas are constructed for collecting maximum amount of power from the radio sources and resolving their features. An antenna with very large physical area can be a solution, but there are physical constraints such as weight, thermal, structural and mechanical problems which impose a limit to the size of antennas. However, it can be shown that the performance of a single large antenna can be achieved by properly combining the outputs of a group or array of antennas. This is in effect the principle of interferometry. An array is made of a number of dipoles or parabolic dishes. A $\lambda/2$ dipole in free space has a directivity of 1.6 compared to an isotropic antenna whose directivity is taken as 1. Thus the directivity of the halfwave dipole is 1.6 or expressed in dB notation, it is 2.15 dBi (the subscript i indicates isotropic antenna taken as reference). For practical purposes, directivity can be taken as the gain of an antenna. Higher gains can be achieved by grouping a number of dipoles as a linear array, with appropriate spacing between them and adding their outputs in phase, resulting in a sinc function response. However simple dipole arrays have directional ambiguity due to the presence of back lobes. So,antennas with reflector and having lot more gain, are used in interferometric arrays. The gain depends upon the aperture or the area intercepting the incoming radiation (projected area). Mathematically, it is described by the equation:

$$G = 4\pi\ A_e/\lambda^2 \tag{3}$$

Thus, in the case of a 4 m dish with 50% efficiency, the gain comes out to be $\approx$ 33 dB.

Unwanted Radiation from sources outside the aperture of the dish could be picked up through the side lobes of the antenna, if the F/D is high and the primary beam of the feed is broad. This raises the system temperature and deteriorates the sensitivity and the S/N (pronounced as signal to noise ratio). We look for an antenna with go figure of merit or G/T (pronounced as G over T). There, T represents the system temperature which is made up of the galactic noise which is quite high at low frequencies (up to about 10 GHz) and by the atmospheric noise at high frequencies and the noise contribution of the devices like the feed, front end electronics and cables and the beam pattern of the dish. In the following section, we will calculate the G/T for the 4 m dish and show how to measure it in practice.

3.1 CALCULATION OF T_{system}

$$T_{system} = T_{ant} + T_{Rx} \tag{4}$$

T_{ant} is essentially due to the average sky background and due to the spillover noise contribution through the side lobes of the beam form Ground and other sources. Let us assume a value of 10 K for the galactic noise in the 1.4 GHz band and 30 K for the spill over noise. Let the noise temperature of the front end system be taken to be 60 K.

$T = 100$ K $\equiv 20$ dB.

Thus,

$(G/T)_{theoretical} =$ (33-20) or 13 dB.

3.2 HOW TO FIND G/T EXPERIMENTALLY?

We can do an experiment for measuring the noise from the cold sky, making sure that there are no strong sources nearby (call this power received as a: $a \propto T_{coldsky}$) and then focus on to Sun, whose flux is known (call this power as b: $b \propto (T_{sun}+T_{coldsky})$). Let us now define a quantity Y,

$$Y = b/a = (T_{sun} + T_{coldsky})/T_{coldsky},$$

rearranging the terms, we get,

$$Y - 1 = T_{sun}/T_{coldsky}$$

$$T_{sun} = (Y - 1)T_{coldsky}$$

$$G = 4\pi\ A_e \div \lambda^2$$

$$1/2SA_e = kT_{sun}$$

$$A_e = 2k\ T_{sun}/S$$

$$G/T_{coldsky} = 8\pi\ k(Y - 1) \div (S\lambda^2 \tag{5}$$

Since $T_{coldsky} = T_{system}$, the above expression gives the required G/T.

In a typical case where S is taken as 1 million J, the measured deflection when antenna is move from cold sky to Sun is 12 dB; the corresponding value of hence Y = 15.84. Using all these values, we can find G/T at $\lambda = 0.2$ m.

$(G/T)_{measured} = 12.9$ (11.1 dB), which agrees fairly well with the theoretical value, within the assumed values of S and other parameters.

4 SYSTEM TEMPERATURE AND NOISE FIGURE MEASUREMENT

In RA, both the signal and noise are of Gaussian random noise type, the difference being that the former is what is picked up from the required radio source thru' the main lobe of the beam while the latter, through the side lobes. In order to make a distinction between the two, we can integrate the incoming radiation for longer periods. It can be shown that by increasing the number of samples (long term integration) and increasing the (pre-detection) bandwidth, the rms fluctuations (ΔP) of a broadband stationary random noise sequence can be reduced to a value below the wanted signal power.

$$\Delta P = P_{mean} \div \sqrt{N} \tag{6}$$

Let the noise bandwidth be Δf, hence the minimum sampling time is: $1/(2\Delta f)$, the total number of samples N made over an observing time (integrating time) t will be:

$$N = 2 \times \Delta f \times t \tag{7}$$

combining the last two equations, we get

$$\Delta P = P_{mean} \div \sqrt{2 \times \Delta f \times t} \tag{8}$$

It can be seen that the sensitivity of the receiver system improves with increasing bandwidth and integration time. Let us calculate and find out what it means for the parameters used above. Given T_{system} = 100 K, BW = 2 MHz and assuming an integration time of 1 second, the sensitivity comes out to be = 50 milli K. This is the minimum detectable signal level or the condition of S/N = 1. The antenna temperature due to the observed source, should have at least this much flux density for detection, under the conditions of the above parameters. Let us now calculate the T_{ant} of a source having a flux density S = 100 J, observed by a 4 m dish (A_e = 6.25 m^2).

$$1/2 \times S \times A_e = k \times T_{ant} \tag{9}$$

substituting the given values, we see that T_{ant} = 226 milli K and the S/N = 5.

The point to be appreciated is the improvement in the (S/N) at the output of the detector, compared to its input. (S/N)$_{input}$ = T_{ant}/T_{system} = (0.226/100), whereas, (S/N)$_{output}$ = 5, as calculated above. This is a typical case of the total power receiver system. It can be seen that the electronic and sky background need to be stable while integrating the signals for long periods.

5 RECEIVER TEMPERATURE

Any system existing above zero degrees kelvin, is a source of noise. Thus, a system at a temperature T_x, generates (thermal) noise due to random fluctuation of electrons within the device, which is directly proportional to the bandwidth $\delta\nu$

$$N_x = k\,T_x\,\delta\nu \tag{10}$$

A noisy two port network is shown Figure 2a. The subscripts i/x/o indicate the input port /device/output port signal (P) and noise (N) powers. The gain of the network is given by g.

$$P_o = g \times [P_i]$$

$$N_o = g \times [N_i] + N_x$$

$$(S/N)_o = g \times [P_i] \div (g \times [N_i]) + N_x$$

$$[S/N_o] = [S/N_i] \div [1 + T_x/T_i], \tag{11}$$

It is customary to represent the noise performance of a receiver system by the parameter - NOISE FIGURE (F).

$$F = (S/N)_i \div (S/N_o)$$

$$F = 1 \div (1 + T_x/T_i)$$

$$T_x = (F - 1) \times (T_i) \tag{12}$$

In the above consideration, the system gain is shown as g. The derivations are equally applicable to attenuators. It can be shown that the noise figure of an attenuator is the loss (L) itself.

$$T_x = (L - 1) \times (T_i)$$

In the following section, the noise performance of a two stage receiver chain is analyzed. It has been shown how the noise figure of the overall system noise figure depends mostly on the noise figure of the first stage (Figure 2b). The main reason for this is that if the first stage gain is high, the sum of the amplified source noise and the added first stage noise will be quite larger than any other added noise of other stages.

The incoming,first and second stage noise powers referred to the respective stage input, and their corresponding values at the output are shown below:

- input noise = T_i and the corresponding output noise is = $T_i\, B_N\, g_{a1} g_{a2}$
- first stage noise = T_{x1} and the corresponding output noise is = $T_{x1}\, B_N\, g_{a1} g_{a2}$
- second stage noise = T_{x2} and the corresponding output noise is = $T_{x2}\, B_N\, g_{a2}$

The overall system noise is the sum of the above three quantities, which is $g_{a1} g_{a2}\ [T_i + T_{x1} + (T_{x2}/g_{a1})]\ B_N$. We arrive at the expression for effective system temperature as:

$$T_e = [T_i + T_{x1} + (T_{x2}/g_{a1})] \tag{13}$$

Let us look calculate T_e for the typical case:

- Given, T_i = 0.1 K, T_{x1} = 30 K, T_{x2} = 100 K and g_{a1} = 100 K
- T_e = 0.1 + 30 + 100/100 = 31.1 K

It is obvious that the first stage plays major role in deciding the over all noise temperature.

If the first stage is a lossy element like a transmisison line or attenuator, then the over all noise temperature can be shown to be: $T_{e1} = (L-1)T_0$ and the total system temerature = $(L-1) \times T_0 + L \times T_{e2}$

6 NOISE FIGURE

Noise figure, F, is defined as the ratio of the Output noise power and the Input noise power. F is unity for a noiseless system. Following the above discussion, the noise figure of a two stage receiver system is: F = output noise power/input noise power.

$$F = [T_i + T_{x1} + (T_{x2}/g_{a1})]/T_i$$

$$F = 1 + [T_{x1} + (T_{x2}/g_{a1})]/T_i$$

$$(F-1)\ T_i = T_{x1} + (T_{x2}/g_{a1})$$

Generally, T_i is taken to be T_0 or 290 K. The above discussion leads us to an important expression namely:

$$T_e = (F-1)\ T_0 \tag{14}$$

An expression of the same pattern as that of temperature, can be derived for noise figure also.

$$F = F_1 + (F_2 - 1)/g_{a1} + (F_2 - 1)/(g_{a1}g_{a2}) + \dots\dots\dots\dots \tag{15}$$

Let us end the session with two examples:

Example (1): A receiver chain consists of two amplifiers, each having a gain and noise figure of (20 dB, 6 dB) and (90 dB, 13 dB), respectively (Figure 2c). The loss in the cable connecting the two devices is 3 dB. What is the overall noise figure and systems temparature.

Using the equation 15, the over all noise figure can be found out. Care need to be taken to convert the logarithmic quantities into numbers.

$$F = 4 + (2 - 1)/100 + (20 - 1)/(100 \times 0.5)$$

$$F = 4.39 \longmapsto 6.4\,dB$$

Example (2): Let a horn antenna be connected to a LNA (gain 26 dB (400), Noise temperature 4 K, by a wave guide of loss 0.4 dB (loss = 1.1)). Let the following stage gain and noise figure be 17 dB and 6 dB respectively. Let us calculate the Over all noise temp and noise figure.

Noise figure of the first stage (waveguide) = 0.4dB (or gain = (1/1.1)).

The noise temperature of the first stage = (1.1 - 1)× 290 K = 29 K.

The noise temperature of the second stage (LNA) = 4 K.

The noise temperature of the third stage is = (4 - 1) 290 K = 870 K.

$T_{overall}$ = 29 + (4 × 1.1) + (1.1 × 870) ÷ 400 = 35.79 K.

It is clear that inspite of using a very low noise amplifier, the loss in the waveguide has increased the overall noise temperature. So, losses in the front end stage is detrimental to the receiver performance.

The overall noise figure = $(1 + T_e)/T_0$ can be found out to be = 0.5 dB.

7 SUMMARY OF FORMULAE

- Brightness B units: Watts/(m$^2\times$ Hz $\times$ sr)
- Power flux density $S = \int\int B(\delta\lambda)$ Watts/m^{2}Hz
- Power flux density S units: Watts /m^2 Hz
- Total power flux density S_T units: Watts/m^2
- Power in terms of temperature $P = kT\delta\nu$ Watts
- Brightness temperature $T = B \times \lambda^2 \div (2k)$ deg K
- Power flux density $S = 2kT/A_e$ Watts/(m^2 Hz)
- Minimum detectable temperature $S_{min} = 2kT_{min}/A_e$ Watts/m^2 Hz
- Minimum detectable temperature $T_{min} = T_{system} \div \sqrt{2B\tau}$ deg K

8 SINGLE TELESCOPE RECEIVER SYSTEM

Usually, a single antenna receiver system consists of stages of RF amplifier, band pass filter, local oscillator, mixer, IF amplifier, IF filter,square law detector and video amplifier. The RF spectrum is converted to the video band in stages, so that the frequency selectivity is high and the system gain of about a million or so, is spread over the different stages. The choice of the front end system depends upon the application. As described above, the front end stage noise figure should be kept low to keep the overall T_{sys} is low and improve sensitivity. Sometimes,it might become necessary to use a lossy, narrowband RF filter before the LNA, to avoid active devices getting saturated by some strong external RFI and produce inter modulation noise. In such situations, one might choose to increase the bandwidth or the integration time or use other techniques like interferometry, to increase the selectivity.

The mixer translates the frequency spectrum from the RF to IF band, using the Lo signals. The amplitude of the Lo signal is sufficiently high to drive the mixer in to nonlinear mode of operation and generate sum, difference products of the LO and the RF spectrum. The mixer output would also contain other frequency components like harmonics of the LO and other inter-modulation products, which are to be filtered out. The IF signal is amplified sufficiently for driving a detector diode. The diode is operated in the square law region, where, *output voltage* $\propto$ *input power*.

A low pass filter at the output of the detector acts as an integrator. This are essentially the components of each antenna receiver system to be used in the BDA.

Commonly, the IF o/p of the receiver is down converted into sine and Cosine outputs, by using two Local Oscillator signals separated by 90 deg. in order to obtain a vectorial output. The sine and cosine outputs of each antenna are correlated independently to obtain fringes. The fringes obtained using the sine and cosine Correlator differ by 90°. and the rate of fringing is directly proportional to the spacing between the pair of antennas. The details will be covered in the separate lecture program.

The above system is referred to as total power receiver system. Since the total power receiver has no mechanism to discriminate between the external signal and the internally generated noise and other instabilities, the system is affected by parameter changes due to temperature, ground pick-ups, power supply variations and so on. The o/p of the system is affected, needing calibration of the system gain. A variation of the total power receiver is a Dickie receiver, wherein, the input is switched alternatively, between a radio source whose amplitude is known and a standard noise source. At the detector end, the levels are compared and system gain and the signal level can be calibrated. One can device a feedback for taking care of the system instability, using this kind of calibration. Since the telescope is looking at the radio source only for part of the time, the overall efficiency of a Dicke receiver is less when compared with the total power receiver.

From the above discussions it is clear that, for a given T_{system}, $\delta\nu$ and integration time τ, sensitivity and angular resolution improve with A_e. However, as the size of the antenna is increased, cost and complexity of the telescope increase due to mechanical and structural considerations.

Alternatively, higher angular resolution can be achieved by using a number of smaller antennas as interferometers, effectively forming a large aperture. Mapping is done by scanning the source like in TV reception, to obtain the spatial frequency components - u and v; many pairs of (u,v) components can be obtained to form an image of the source. In the following sections, we will discuss about interferometer system properties.

9 RADIO INTERFEROMETERS

The need for high resolving power led to the development of radio interferometers, using arrays of antennas. Similar to the case in optics, the output of each antenna, can be made to interfere with the other and if the path delay between them is suitably adjusted, signals can be made to reinforce on one another. one can correlate either the voltage inputs as in the conventional interferometer or the power, as in the intensity interferometer. We will deal with the voltage multiplier type in this lecture program and briefly mention about the intensity interferometer at the end of this section. The multiplied o/p represents the coherence or otherwise between the pair of signals, for a given baseline and position of the source. For a given wavelength λ and spacing D between a pair of antennas, the angular resolution is $\propto (\lambda \div D)$, which is much higher than that achievable by individual dishes. The principle of continuoulsy combining the signals of the set of antennas by adjusting the relative path delay, is called aperture synthesis (Figure 3).

Due to the relative motion between the observer and the source, the time of arrival of a wavefront at the antennas differs and it with the rotation of the earth and the position of the source. To illustrate the point, let us assume a pair of antennas looking at a source that is at Zenith. Under this condition ($\theta = 0$), a wavefront arrives at the a pair of antennas, without any time or path difference. The geometric delay between them is zero. As the earth rotates, the PROJECTED BASELINE between the antennas varies and so the geometric delay of the wave front arriving at one antenna, leads or lags behind the other. If the received signals are then multiplied and integrated over the band width of the incoming signal, the output is said to fringe in a (quasi) sinusoidal fashion as the delay varies (see details of Fringe delay calculations for WSRT in the website www.nfra.nl). The delays can be compensated by correcting the phases in a correlator and high resolution mapping can be obtained by correlating the amplitude and phase fringe pattern of pairs of antennas. The interference pattern represents the Fourier Transform of the brightness function. Inverse Fourier Transform gives the image of the source.

The technique os using an array of antennas is called aperture synthesis interferometry. Using the rotation of the Earth, the multiple base lines of the interferometric antennas will scan the source in all possible orientations and trace out ellipses and map the source. The resolution for a given baseline D is given by:

$$\text{Resolution } R = 1.22 \ (\lambda / D \ \sin\theta) \tag{16}$$

It can be verified that at cm wavelengths, a baselines of a few meters to a km yield significant resolutions between a few hundred milli arc seconds to a few arc seconds. The functional equation for an interferometer can be shown to be:

$$F = \cos(2\pi \ D \ \sin(\theta) \div \lambda) \tag{17}$$

This is the representation of the fringe function F, at a single frequency. Since the incoming signal consists of a band of frequencies, the periods of the individual functions are different form each other which can be described as a kind of beat or modulation pattern. The over all fringe for the band of frequencies can be described as that corresponding to a single frequency modulated by a Sinc function, as shown below:

$$F(\nu_0) = \cos(2\pi \ D \ \sin\theta \ \nu_0/c) \ sinc(\pi \ D \ \sin\theta \ \ \Delta\nu/c) \tag{18}$$

The fringe frequency and the rate of fringing can be calculated for the interferometer in the following manner (see the website www.nfra.nl). The path delay in terms of the HA and Dec, is:

$$path \ \ delay = D * \cos(Dec) * \sin(HA) \tag{19}$$

$$phase \ \ delay = D * \cos(Dec) * \sin(HA) * \nu_0/c \tag{20}$$

$$fringe \ \ frequency \propto D * \cos(Dec) * \cos(HA) * \nu_0/c \tag{21}$$

For the conditions of $\nu_0 = 1.4$ GHz, zero deg. Dec and HA, $D = 8$ meters, the fringe frequency comes out to be: 280×10^{-5} Hz or a period of about 6 minutes of time. Figure 5. shows the fringe patterns for the various spacings of the BDA array.

The response of an array to radio frequency interference (RFI) is a favourable factor. The spectra of the RFI signals differ with that of the interferometers. The relative delays of the signals received by widely spaced antennas are quasi-indeterministic and lack coherence.

Let us now briefly discuss the property of the Correlator receiver. It consists of a multiplier and an integrator and estimates the amount of correlation between a pair of signals. let us call the input voltage contribution by each antenna as $v1$ and $v2$.

- $v1 \propto \sqrt{(Ta1 + Ts1)}$
- $v2 \propto \sqrt{(Ta2 + Ts2)}$

The normalized cross correlation coefficient is expressed as:

- $\rho = (v1 \times v2^{*}) \div \sqrt{(v1^{2} \times v2^{2})}$

After substitution by the temperature terms, the normalized cross correlation coefficient becomes:

$$\rho \propto \sqrt{Ta1 \times Ta2 \div [Ta1 + Ts1] \times [Ta2 + Ts2]} \qquad (22)$$

The rms noise at the correator o/p contains noise contribution of both the terms. Using Nyquist sampling condition, it can be represented by the expression:

$$\Delta\rho = 1 \div \sqrt{2\Delta\nu t} \qquad (23)$$

Using the above equations, let us find the S/N (= $\rho/\Delta\rho$), of the Correlator, under the assumption $Ta << Ts$.

$$S/N \propto \sqrt{[Ta1Ta2/Ts1Ts2] \times (2\Delta\nu t)} \qquad (24)$$

The equation is an important one and often used for calibration of the Correlator system sensitivity (S/N = 1 or as defined by each user) of the instrument.

The above equation can be expressed in terms of the apertures. Thus

$$S_{min} \propto \sqrt{[Ts1Ts2]/[A1A2(2\Delta\nu t)]}. \qquad (25)$$

Rigorous treatment of sensitivity can be found in the website www.nfra.nl, chapters 6 and 7, pages 155-158 and 139-148, respectively. Because the interferometer does not make use of the information in the self correlation part of the signal, the sensitivity can be shown to be poor by a factor of $\sqrt{2}$ for weak sources and the same for strong sources, when compared with a total power receiver, under same bandwidth,integration time, area of the antenna and T_{system} conditions.

10 INTENSITY INTERFEROMETER

In this type, the power in each arm of the pair is detected before applying delay compensation to one of the arms, and multiplying. The correator o/p is $\propto <(v1^{2}v2^{2})>$, which can be simplified to $<(v1v2)^{2}>$, the squared modulus of the complex visibility of the source. Since it responds only to the amplitude, the o/p of the detector does not represent the frequency and phase characteristics of the input signals. The Correlator o/p does not show any fringes. The degree of coherence between the two signals is proportional to the integrated spectral power. The effect of ionospheric phase irregularities are absent which is an advantage for long baseline interferometry, but the overall sensitivity of the system is poor compared to the voltage multiplier type Correlator. This is not helpful for mapping the complex visibility function of the source.

11 PHASED AND CORRELATOR ARRAY

Two modes of configurations are used in interferometry (Thompson et al., 1991, pages 121-123). These are the phased array and Correlator modes. In the former, phased shifters are present in each antenna line for delay adjustment before combining them in a kind of X-mas tree fashion. The o/p of the array is the square of the sum of the voltages of each antenna. The beam of the array can be steered in a sinusoidal manner, by controlling the phase shifts (Figure 4a).

In the correator array mode of operation, $n(n-1)/2$ combinations of the signal products are obtained simultaneously (equivalent to simultaneous formation of beams) from n antennas and correlated. These o/p of the correator channels represent the self and cross correlation products in the form of V_m^2 and $V_m \times V_n$, respectively. The self o/0/p s are used for individual antenna calibration and the cross products for estimating the coherence amongst the signals in frequency and spatial domains (Figure 4b).

12 GROUNDING AND EARTHING

The most bothersome factor in radio astronomy is the noise generated internally and externally to the system. A reduction of noise to reach the thermal limit of noise is the desire of all astronomers. Cooling the front end receivers is desirable to keep t he system noise low. Other internal noise sources like due to inter modulation products, LO leakage are to be kept in mind while designing the system. Components like LNAs and mixers are to be designed to be capable of handling smallest to largest range of signals (dynamic range). The leakages from power supply, coupling due to leaky transmission lines, impedance mismatch between units, radiation from improperly shielded RAF and digital stages, power supply lines etc. could cause enormous damage to the quality of reception.

Interconnection between the systems should be such that the return wires of the power supply of individual systems are connected to a reference ground independently (like the branches of tree).

Another worrisome factor is the damage due to lightning. If the signal ground(s) and the electrical grounds are mixed up, it is possible that when a lightning strike occurs, heavy currents returning to mother ground get diverted to the electronic system and cause damages. It is advised to have separate signal and electrical grounds. The resistance of the grounds need to be monitored and maintained regularly. Surge suppressors and Metal Oxide Varistor capacitors need to be introduced for surge protection, particularly for the front end systems. The antennas need to be grounded well.

13 REFERENCES

Birney, S. D. **Observational Astronomy**. Cambridge, UK: Cambridge University Press, 1991. 321 p.

Handbook on Radio Astronomy, International Telecommunications Bureau: Geneva, 1995.

Thompson, A. R.; Moran, J. M.; Swenson, G. W., Jr. **Interferometry and synthesis in Radio Astronomy**. Melbourne, FL, USA: Krieger Publishing Company, 1991. 556 p.

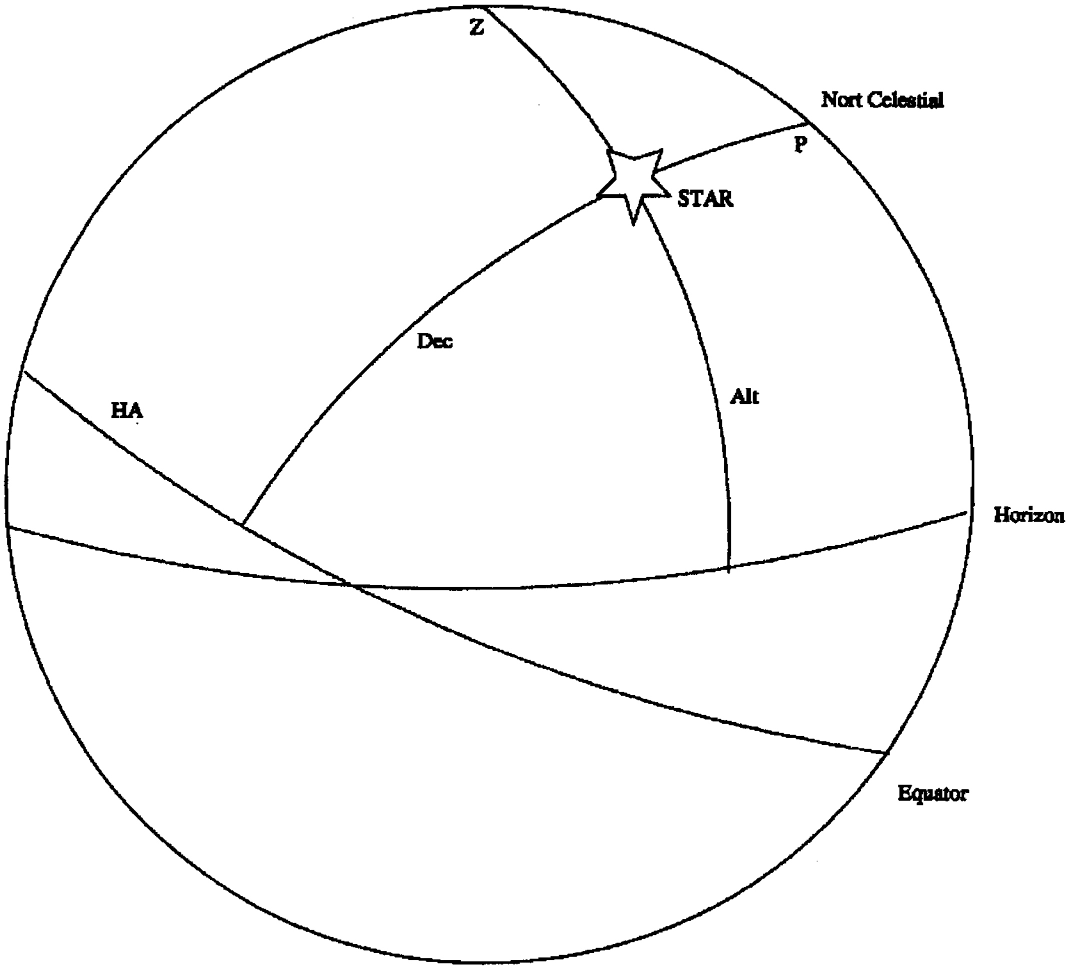

Figure 1: Representation of the position of the radio source in the celestial sphere.

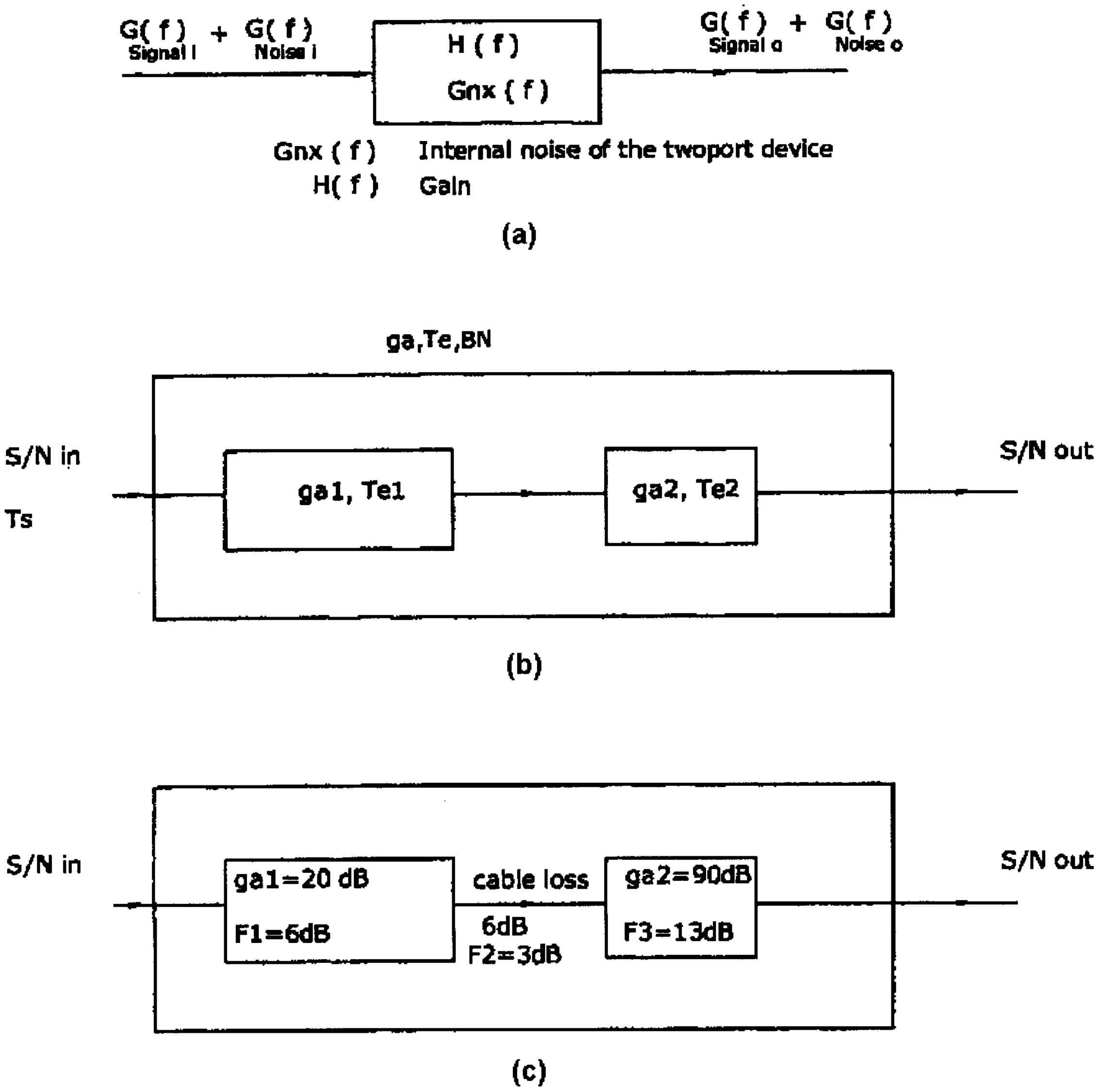

Figure 2: (a) Noisy two port network. The subscripts i/x/o indicate the input port /device/output port signal (P) and noise (N) powers. (b) Diagram showing how the noise figure of the overall system depends mostly on the noise figure of the first stage. (c) example of a receiver chain consists of two amplifiers, each having a gain and noise figure of (20 dB, 6 dB) and (90 dB, 13 dB), respectively.

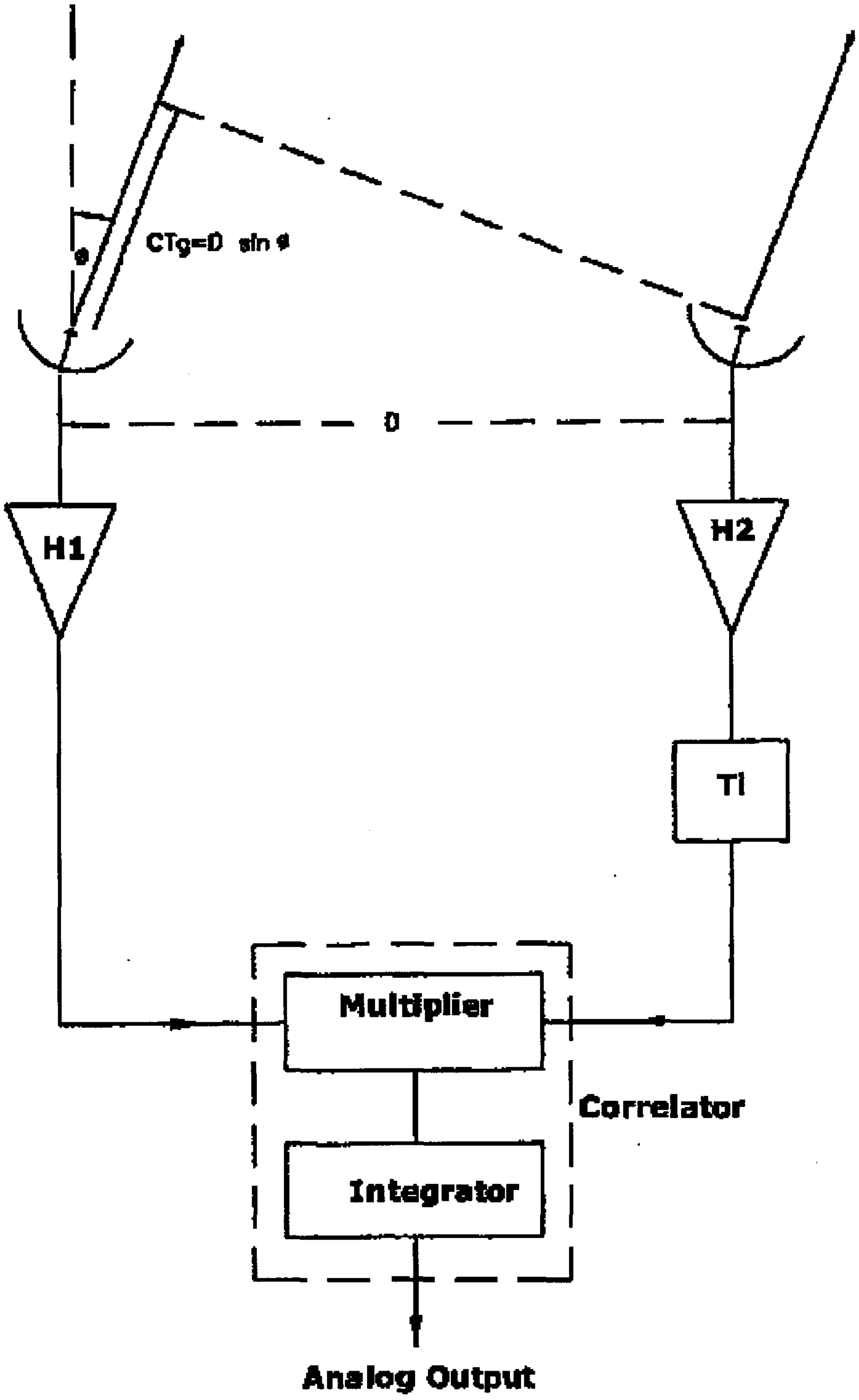

Figure 3: Analog Correlator Receiver.

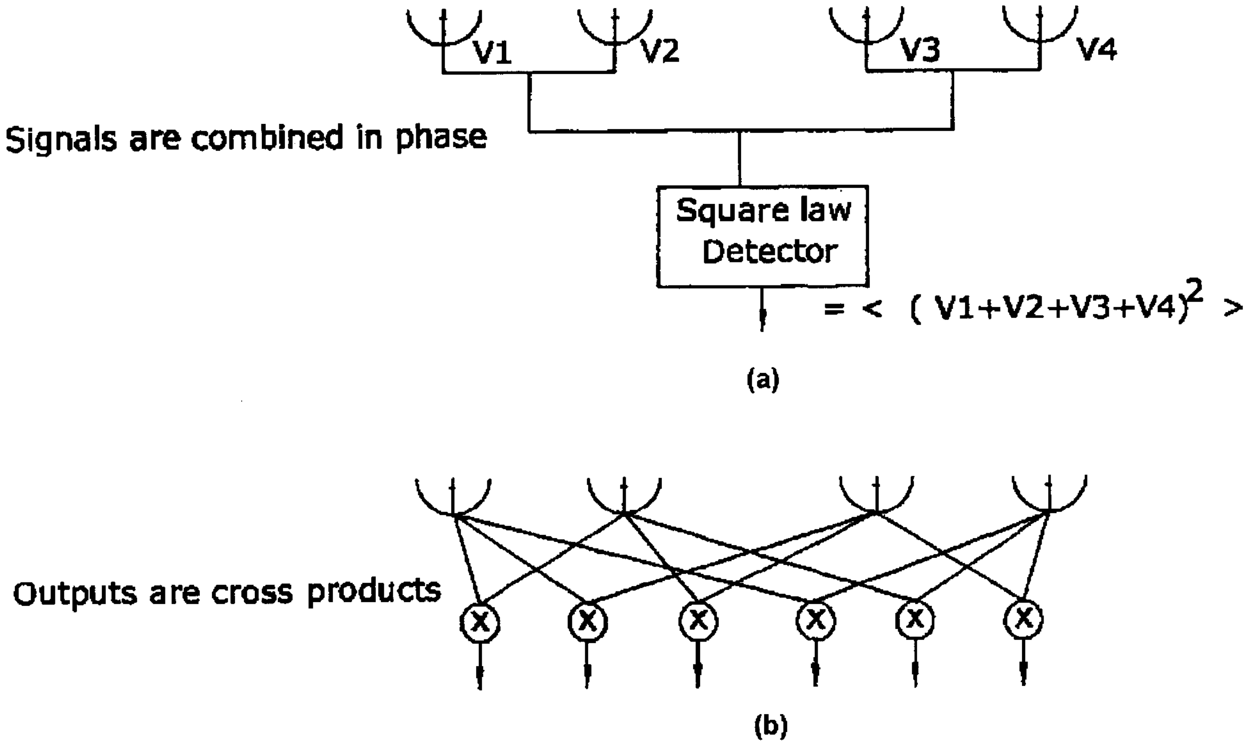

Figure 4: (a) Principle of phased array. (b) Principle of correlator array.

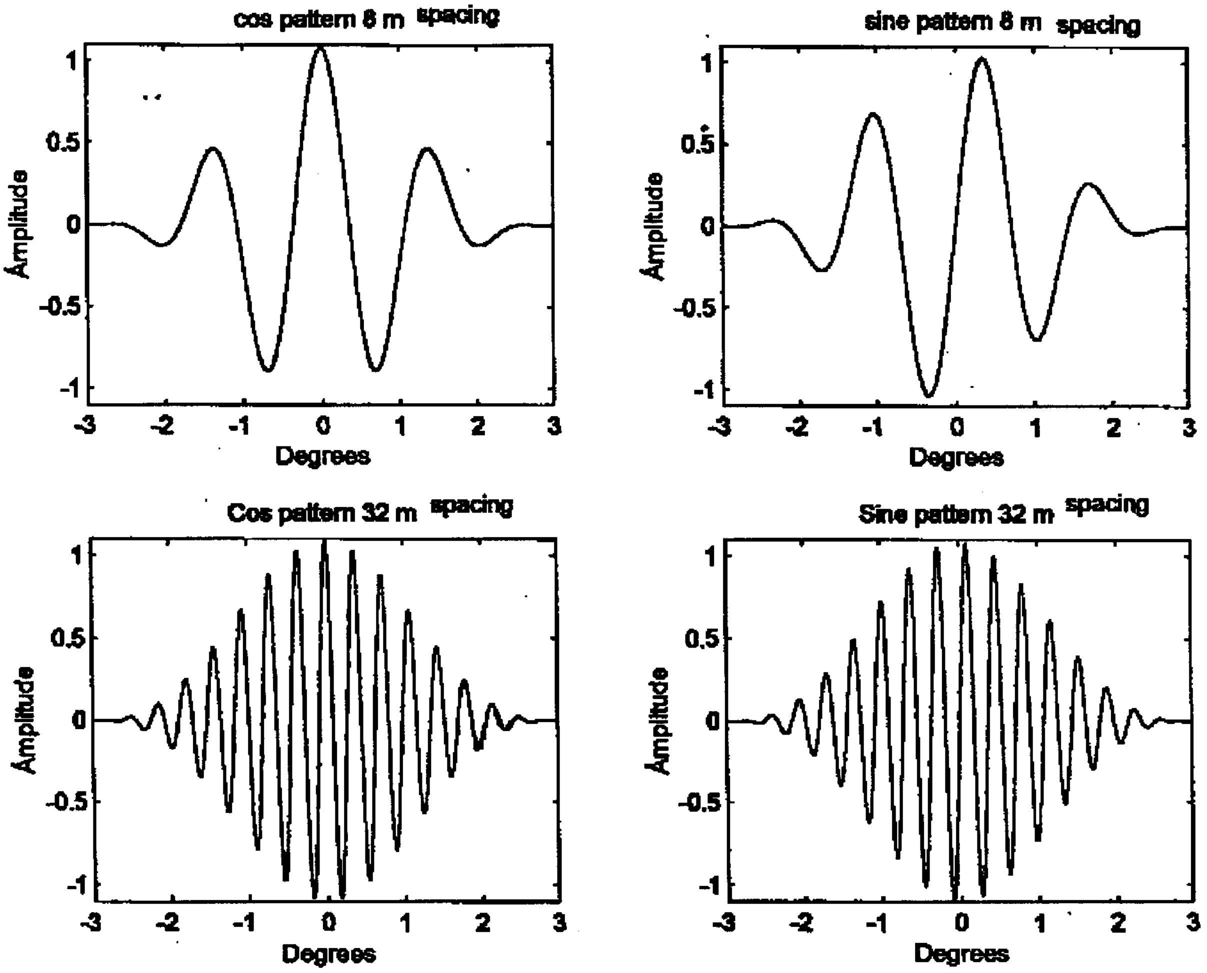

Figure 5: Fringe patterns of BDA for 2 elements with spacing of 8 m and 32 m.